GOVERNMENT DEGREE COLLEGE

VEERAGHATTAM

DEPARMENT OF MATHEMATICS

GUEST LECTURE RECORD :2024-25

BY

CH NAIDU

LECTURER IN MATHEMATICS

From

The Principal

Govt Degree College, Palakonda,

Parvathipuram-Manyam(Dt)

To

The Principal

Govt Degree College, Veeraghattam

Parvathipuram-Manyam(Dt)

Sir

Subject: GDC-Palakonda-Request to depute Sri Ch.Naidu, Lecturer in Mathematics for guest lecture on 08.04.2025-Reg.

It is to bring to your kind notice that the Department of Mathematics of our college has planned to organize a guest lecture on 08-04-2025. Hence I request you to depute Sri Ch.Naidu , Lecturer in Mathematics GDC, Veeraghattam for delivering guest lecture at GDC-Palakonda, Parvathipuram Manyam (Dt).

This is for your kind information.

Thanking you Sir,

PRINCIPAL
GOVT. DEGREE COLLEGE
PALAKONDA
Parvathipuram Manyam Dist.

GOVT.DEGREE COLLEGE, VEERAGHATTAM, PARVATHIPURAM MANYAM DIST

ATTENDANCE RELIEVING CERTIFICATE

This is to certify that Sri Ch.Naidu, Lecturer in Mathematics, Govt.Degree College, Veeraghattam has been relived from his duties on 07-04-2025 F.N to attend Guest Lecturer on 08-04-2025as per request letter GDC Palakonda.

Principal 7/4/25

Principal
Govt. Degree College
VEERAGHATTAM

TOPIC: REAL ANALYSIS

Basic concept of real analysis:

Real valued functions: *If the Range of function is subset of $\mathbb{R}$ or $\mathbb{R}$ itself then it is called as real valued function*

i.e. if X is any set, $f: X \longrightarrow \mathbb{R}$ then f is called as real valued function

Real function: *If both Domain and Range is subset of $\mathbb{R}$ or $\mathbb{R}$ itself then it is called as is called a real function. i.e. if $X = \mathbb{R}$, $f: X \longrightarrow \mathbb{R}$ then f is called as real function*

Constant function: A function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = k(k \in \mathbb{R})$ is called a constant function.

Polynomial function: A function $f: \mathbb{R} \to \mathbb{R}$ defined by
$f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$

where $a_0, a_1, a_2, \ldots a_n \in \mathbb{R}, n \in \mathbb{R}$ *and* $a_n \neq 0$ is called a polynomial function of degree n

Absolute value function (or) Mod functions: -

The function $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x$ *if* $x \geq 0$ *and* $f(x) = -x$ *if* $x < 0$ is called mod function and it is denoted by $f(x) = |x|$

1. **Modulus of a real number:** *If x is a real number then the modulus of x is denoted by $|x|$ and is defined as* $|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$

Ex: $|-5| = -(-5) \quad \because x = -5 < 0$

$= 5$

2. $|x| < \delta \Leftrightarrow -\delta < x < \delta$

3. $0 < |x| < \delta \Leftrightarrow 0 < x < \delta \text{ and } -\delta < x < 0$

$\Leftrightarrow x \in (-\delta\ 0) \cup (0\ \delta)$

4. $|x - a| < \epsilon \Leftrightarrow -\epsilon < x - a < \epsilon$

$\Leftrightarrow a - \epsilon < x - a + a < a + \epsilon$

$\Leftrightarrow a - \epsilon < x < a + \epsilon$

5. $0 < |x - a| < \epsilon \Leftrightarrow a - \varepsilon < x < 0 \text{ and } 0 < x < a + \epsilon$

6. $|xy| = |x||y| \text{ and } \left|\frac{x}{y}\right| = \frac{|x|}{|y|} \text{ if } y \neq 0$

7. $|x + y| \leq |x| + |y|$

8. $|x - y| \geq |x| + |y| \text{ or } |x| + |y| \leq |x - y|$

9. $|x - y| \geq \big||x| - |y|\big| \text{ or } \big||x| - |y|\big| \leq |x - y|$

10. ***Integral part of a real number***: If x is real number then the integral part of x is denoted by $[x]$ and it is defined as integral n such that $n \leq x < n+1$.

$x-[x]$ is called as fractional part of x

Ex: 1) $[3.1] = 3$ since $3 < 3.1 < 4$ 2) $[3] = 3$ since $3 \leq 3 < 4$

3) $[3.5] = 3$ since $3 < 3.5 < 4$ 4) $[2.9] = 2$ since $2 < 2.9 < 3$

5) $[-3.5] = -4$ since $-4 < -3.5 < -3$

Note: 1. The integral part of any real number other than integer is the immediate integer to the left side

2. The integral part of any real number x defined as the greatest integer not greater than x.

Neighbourhod of a point: Let $P \in \mathbb{R}$ and $\epsilon > 0$ then the set $\{x/|x-p| < \epsilon\}$ is called as $\epsilon -$ nbd of p i.e. $\epsilon -$ nbd of $p = \{x/|x-p| < \epsilon\}$

$$= \{x/p-\epsilon < x < p+\epsilon\}$$

$$= (p-\epsilon \ \ p+\epsilon)$$

Deleted Neighbourhod of a point: The set $\{x/|x-p| < \epsilon \text{ and } x \neq p\}$ is called as deleted $\epsilon -$ nbd of p i.e. $\{x/0 < |x-p| < \epsilon\}$

Deleted nbd of a point $p = \{x/0 < |x-p| < \epsilon\} = (p-\epsilon \ \ p) \cup (p \ \ p+\epsilon)$ and it is denoted by $\mathcal{N}_\epsilon(p) - \{p\}$.

Limit point: A point $p \in \mathbb{R}$ is said to be limit point of a subset S of $\mathbb{R}$ if every nbd of p has infinitely many points of S. i.e. $p \in \mathbb{R}$ is a limit point of S if $\forall \epsilon > 0$ such that $(p-\epsilon \ \ p+\epsilon) \cap S =$ infinite set (OR)

A point $p \in \mathbb{R}$ is said to be limit point of a subset S of $\mathbb{R}$ if every nbd of p has a point of S other than P itself.

Upper bound: Suppose S is a subset of $\mathbb{R}$ and $a \in \mathbb{R}$. If $x \leq a \ \forall x \in S$ then 'a' is called upper bound of S. S is said to be bounded above if S has an upper bound.

Ex: 1. $\mathbb{R}^- = (-\infty \ \ 0)$ is bounded above and 0 is upper bound

2.. $\mathbb{R}^+ = (0 \ \ \infty)$ is not bounded above because it has no upper bound.

3. Every finite set is bounded above. Ex: $S = \{1,2,3,4\} \subseteq \mathbb{R}$

Least upper bound (l.u.b) or supremum: If u is an upper bound of S and v is an upper bound of implies $u \leq v$. Then u is said to be a least upper bound of S

$(or) l.u.b$ (or) $supremum of$ S.

Lower bound: Suppose S is a subset of $\mathbb{R}$ and $a \in \mathbb{R}$. If $x \geq a$ $\forall x \in S$ then a is called lower bound of S. S is said to be bounded below if S has an lower bound.

Ex: 1. $\mathbb{N} = \{1,2,3, \ldots\ldots\}$ is bounded below and 1 is lower bound of $\mathbb{N}$

2. $\mathbb{R}^- = (-\infty\ 0)$ is not bounded below because it has no lower bound

2.. $\mathbb{R}^+ = (0\ \infty)$ is bounded below and 0 is lower bound of $\mathbb{R}^+$

Greatest lower bound $(g.l.b)$ or Infimum: If v is a lower bound of S and u is a lower bound of S implies $u \leq v$ then v is said to be greatest lower bound $(or) g.l.b$ (or) infimum of S

Bounded set: An aggregate S is said to be bounded if it is both boundd below and bounded above. i.e. S is bounded $\Leftrightarrow \exists\, v, u \in \mathbb{R}$ such that $v \leq x \leq u$ $\forall x \in S$

or S is bounded $\Leftrightarrow \exists\, k \in \mathbb{R}^+$ such that $|x| < k$ $\forall x \in S$

Ex: 1. $S = \{2,4,6,8,10,12\}$ is bounded $\therefore$ Inf of $S = 2$, Sup of $S = 12$

2. $S = \{x/1 < x < 2\}$ Inf $S = 1$, Sup $S = 2$ But they are not elements of S

3. $S = \left\{\frac{1}{n}/n \in \mathbb{N}\right\} = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots\ldots\right\}$ Sup $S = 1$, Inf $S = 0$

4. Find inf and sup of $S = \left\{1 + \frac{(-1)^n}{n}/n \in \mathbb{N}\right\}$

Sol: Given $S = \left\{1 + \frac{(-1)^n}{n}/n \in \mathbb{N}\right\} = \left\{0, \frac{3}{2}, \frac{2}{3}, \frac{5}{4}, \ldots\ldots, 1\right\}$

Inf $S = 0$, $\sup S = \frac{3}{2}$

5. Find inf & sup of $S = \{x \in \mathbb{R}^+/x^3 < x\}$

Sol: Given $S = \{x \in \mathbb{R}^+/x^3 < x\}$

$$= \{x \in \mathbb{R}^+/x^3 - x < 0\}$$

$$= \{x \in \mathbb{R}^+/x^2 - 1 < 0\}$$

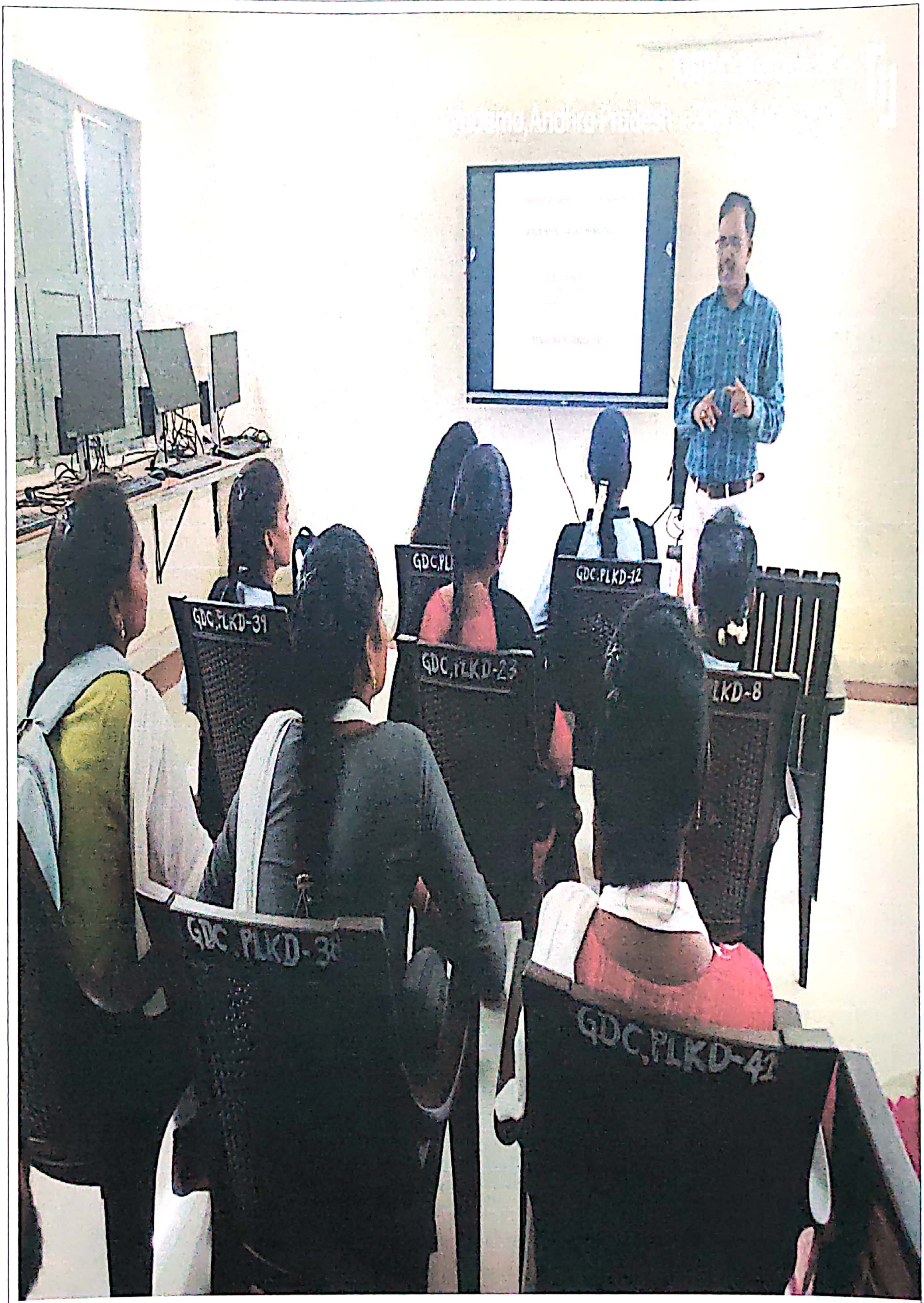
$$= \{x \in \mathbb{R}^+/x \in (-1\ 1)\}$$

$$= \{x \in \mathbb{R}^+/x \in (0\ 1)\}$$

$\therefore \sup S = 1$, $\inf S = 0$

Well ordered principal: Every non – empty set of $\mathbb{Z}^+$ has a least element.

Dense property: Between any two distinct real numbers there exist a rational (or) irrational number.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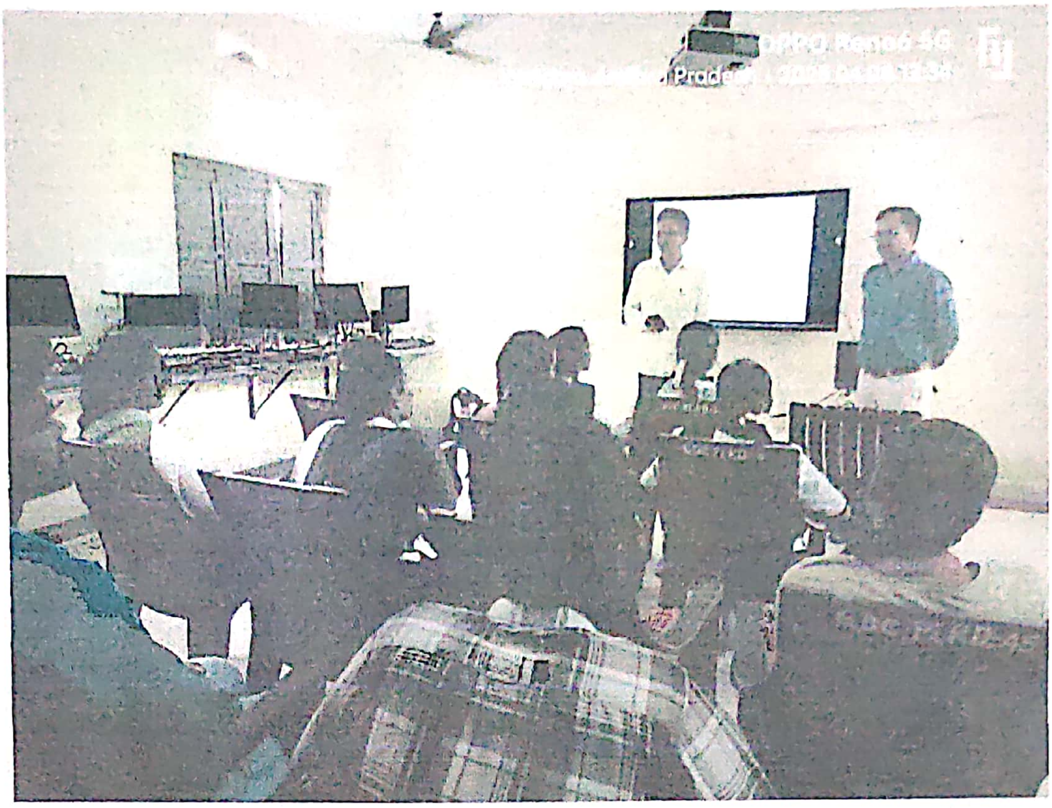

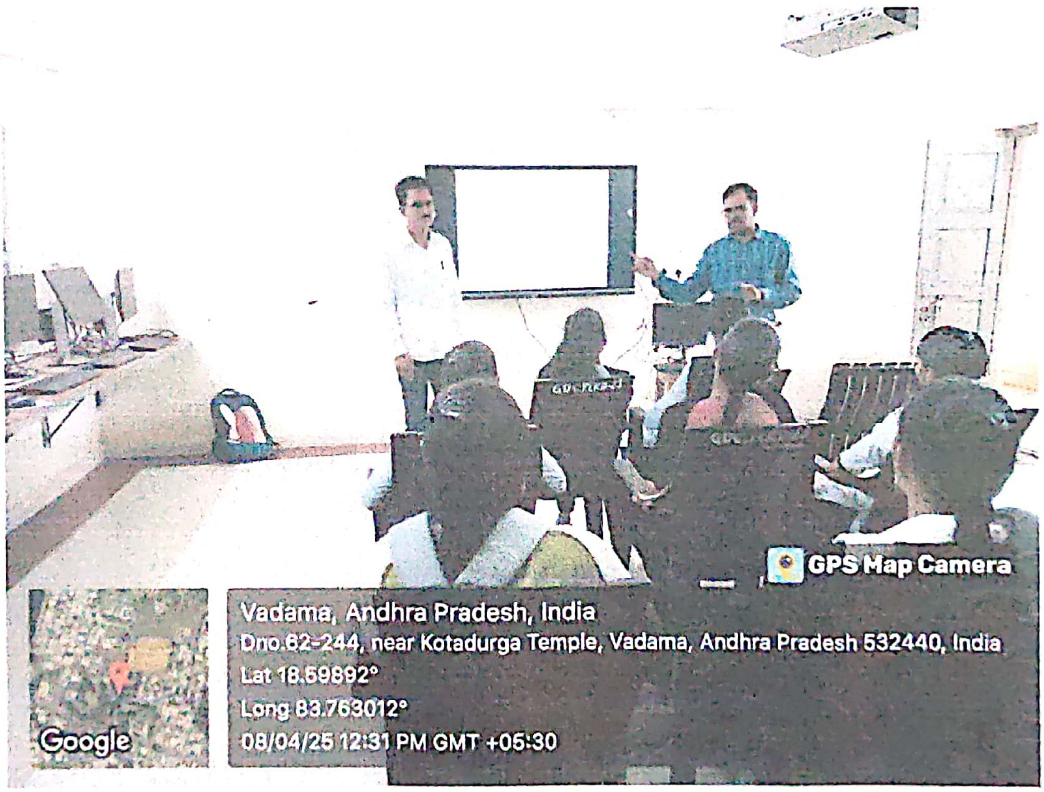

Guest Lecture.

TOTAL NUMBER OF STUDENTS WERE PARTICIPATED IN SEMENOR TOPIC ON REAL ANALYSIS:10

S.NO	NAME OF THESTUDENT	SIGNATURE OF THE STUDENT
1	I.DHANA LAKSHMI	Dhanalakshmi.I
2	P.ANKITHA	P Ankitha
3	L.GAYATRI	L. Gayatri
4	R.BANGARUTALLI	R. Bangaruthalli
5	CH .JYOTHI	Ch. Jyothi
6	S.PRAMEELA	S. PRAMEELA
7	V.VARALAKSHMI	V. Varalakshmi
8	P.SRAVANI	P. Sravani
9	K. HARIBABU	K. HariBabu
10	K.DHILLESWARARAO	K. Dhilleswararao

8/4/2025

SIGNATUE OF DEPARMENT INCHARGE

(Dr CH Raghavendra Naidu)

LECTURER
GOVT. DEGREE COLLEGE
PALAKONDA
PARVATHIPURAM MANYAM DIST

GOVERNMENT DEGREEE COLLEGE
PALAKONDA - 532440
(Affiliated to Dr.B.R.Ambedkar University, Srikakulam)
Parvathipuram Manyam District, Andhra Pradesh

Attendance Certificate

This is to certify that Sri Ch.Naidu, Lecturer in Mathematics, Government Degree College, Veeraghattam, has attended our college and delivered a guest lecture on Real Analysis on 08-04-2025.

He pleased all the students with his outstanding presentation.

Palakonda,

Date:08-04-2025

8/4/25

Principal

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GOVT. DEGREE COLLEGE
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Parvathipuram Manyam Dist.